Homework #5 of Topology II
Due Date: March 9, 2018

1. Describe all covering spaces of 2-torus, up to equivalence.

2. Describe all covering spaces of Klein bottle, up to equivalence.

3. Find all covering spaces of $\mathbb{R}P^2 \vee \mathbb{R}P^2$.

4. For a path-connected, locally path-connected and semilocally simply connected space X, call a path-connected covering space $\tilde{X} \to X$ *abelian* if it is normal and has abelian deck transformation group. Show that X has an abelian covering space that is a covering space of every other abelian covering space of X, and that such a "universal" abelian covering space is unique up to isomorphism. Describe this covering space explicitly for $X = S^1 \vee S^1$ and $X = S^1 \vee S^1 \vee S^1$.

5. Given a covering space action of a group G on a path-connected, locally path-connected space X, then each subgroup $H \subset G$ determines a composition of covering spaces $X \to X/H \to X/G$. Show:

   (a) Every path-connected covering space between X and X/G is isomorphic to X/H for some subgroup $H \subset G$.

   (b) Two such covering spaces X/H_1 and X/H_2 of X/G are isomorphic iff H_1 and H_2 are conjugate subgroups of G.

   (c) The covering space $X/H \to X/G$ is normal iff H is a normal subgroup of G, in which case the group of deck transformations of this cover is G/H.